A New Method for Ranking of Intuitionistic Fuzzy Numbers

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ABSTRACT *In this paper, magnitude of membership function and non-membership function of a Trapezoidal Intuitionistic Fuzzy Number (TIFN) are introduced and a new method for ranking of TIFN based on the magnitude is proposed. Numerical examples are provided to illustrate the new ranking method and compared with different existing approaches.*

1.Introduction

Zadeh[1] introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life. The concept of Intuitionistic Fuzzy Set (IFS) [4,6] can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. The IFSs were first introduced by Atanassov[4] which is a generalization of the concept of fuzzy set[1]. Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. Fuzzy numbers must be ranked before an action is taken by a decision maker. Real numbers can be linearly ordered by the relation $\leq$ or $\geq$, however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists. The concept of ranking function for comparing normal fuzzy numbers is compared in Jain[2]. In Mitchell[13] and Nayagam[17] some methods for ranking of IFNs were introduced. Grzegrorzewski [12] suggested the method of ranking IFNs and an ordering method for IFNs by using the expected interval of an IFN. Based on the characteristic value for a fuzzy number introduced in Kuo-Ping Chiao [10], an ordering method for IFN is proposed by Hassan Mishmast Nehi [20].

Fuzzy set theory has been applied to many fields of Operations Research. The concept of Fuzzy Linear Programming was first formulated by Zimmermann [3]. Fuzzy Linear Programming Problems have an essential role in fuzzy modeling which can formulate uncertainty in actual environment. Afterwards, many authors ([7],[8],[9],[14],[15], [16] [19]),considered various types of the Fuzzy Linear Programming Problems and proposed several approaches for solving these problems.

In this paper, magnitude of membership function and non-membership function of a Trapezoidal Intuitionistic Fuzzy Number (TIFN) are introduced and a new method for ranking of TIFN based on the magnitude is proposed. Numerical examples are provided to illustrate the new ranking method and compared with different existing approaches.

Here, in Section 2, some necessary concepts of fuzzy sets and IFS [3,4, 9] are reviewed. In Section 3, a new method for ranking of TIFNs is proposed. In Section 4, numerical examples are provided to illustrate the new ranking method and compared with existing approaches proposed by Hassan [20] and Grzegrorzewski[12]. In Section 5, Conclusion is presented.

2.Preliminaries

Definition 2.1 : Let X be the universal set. An IFS A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x))/x \in X \}$$ where the functions, $\mu_A, \nu_A : X \rightarrow [0,1]$ are functions such that

$$0 \leq \mu(x) + \nu(x) \leq 1 \ \forall x \in X.$$

For each the numbers $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non- membership of the element $x \in X$ to the set, which is a subset of X, respectively.

Definition 2.2 : An IFS $A = \{(x, \mu_A(x), \nu_A(x))/x \in X \}$ of the real line is called an IFN if

(a) A is IF-normal,
(b) A is IF-convex,
(c) μ_A is upper semi continuous and is lower semi continuous.
(d) $A = \{x \in X / \nu_A(x) < 1\}$ is bounded.

Definition 2.3 : The α–cuts of a IFN are a non fuzzy sets defined as

$$(A^+)_\alpha = \{x \in R / \mu_A(x) \geq \alpha\} = A_\alpha \ ,$$

$$(A^-)_\alpha = \{x \in R / 1 - \nu_A(x) \geq \alpha\} = \{x \in R / \nu_A(x) \leq 1 - \alpha\} = A^{1-\alpha}$$

If the sides of the fuzzy numbers are strictly monotone then the convention that

$$f_A^{-1}(\alpha) = A_L^+(\alpha), \ g_A^{-1}(\alpha) = A_U^+(\alpha), h_A^{-1}(\alpha) = A_L^-(\alpha), \text{ and } k_A^{-1}(\alpha) = A_U^-(\alpha)$$

In particular if the decreasing functions g_A and h_A and increasing functions f_A and k_A be linear then we will have the TIF numbers.

Definition 2.4 : A is a TIFN with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ and denoted by $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$. In this case,

$$\mu_A(x) = \begin{cases} 0 & if \ x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & if \ a_1 < x < a_2 \\ 1 & if \ a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4} & if \ a_3 < x < a_4 \\ 0 & if \ a_4 \leq x \end{cases} \quad and \quad \nu_A(x) = \begin{cases} 0 & if \ x \leq b_1 \\ \frac{x-b_1}{b_2-b_1} & if \ b_1 < x < b_2 \\ 0 & if \ b_2 \leq x \leq b_3 \\ \frac{x-b_4}{b_3-b_4} & if \ b_3 < x < b_4 \\ 0 & if \ b_4 \leq x \end{cases}$$

(1)

If in a TIFN A, we let $b_2 = b_3$ (and hence $a_2 = a_3$) then we will give a Triangular Intuitionistic Fuzzy Number (TrIFN) with parameters $b_1 \leq a_1 \leq b_2 (a_2 = a_3 = b_3) \leq a_4 \leq b_4$ and denoted by .

3. New ranking method for TIFNs

Based on the magnitude of a fuzzy number introduced in Abbasbandy [18] we proposed an ordering method for intuitionistic fuzzy numbers.

For an arbitrary intuitionistic fuzzy number $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$, we define the magnitude of membership and non-membership function for intuitionistic fuzzy number denoted by $Mag(A_\mu)$, $Mag(A_\nu)$ respectively, as

$$Mag(A_\mu) = \frac{1}{2}\left(\int_0^1 (f_A^{-1}(r) + g_A^{-1}(r) + a_2 + a_3) f(r)dr\right) \quad (2)$$

$$\text{and } Mag(A_\gamma) = \frac{1}{2}\left(\int_0^1 (h_A^{-1}(r) + k_A^{-1}(r) + b_2 + b_1) f(r)dr\right) \quad (3)$$

where the function f(r) is a regular reducing function on [0,1] with f(0)=0, f(1)=1 and $\int_0^1 f(r)dr = \frac{1}{2}$. Obviously function f(r) can be considered as a weighting function. In actual applications, function f(r) can be chosen according to the actual situation. In this paper we use f(r) = r. In particular, let $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ be a TIFN with membership and non-membership functions, defined as in equation (1) . In this case we have

$$Mag(A_\mu) = \frac{1}{12}(a_1 + 5a_2 + 5a_3 + a_4) \quad \text{and} \quad Mag(A_\gamma) = \frac{1}{12}(b_1 + 5b_2 + 5b_3 + b_4) \quad (4)$$

Now, an ordering could be given on TIFNs as is shown in the following algorithm

Algorithm 4.1 :

As a ranking method, we compare two TIFNs A and B using the following steps :

Step1 : Compute $Mag(A_\mu)$ and $Mag(B_\mu)$ using (2). Then compare them. If they are equal, then go to the step 2. Otherwise rank A and B according to the relative position of $Mag(A_\mu)$ and $Mag(B_\mu)$.

Step2 : Compute $Mag(A_\gamma)$ and $Mag(B_\gamma)$ using (3). Then compare them. If they are equal, then A and B are equal. Otherwise rank A and B according to the relative position of $-Mag(A_\gamma)$ and $-Mag(B_\gamma)$.

Therefore for any two TIFNs A and B $\in$ I, we define the ranking of A and B by the Mag(.) on I as follows:

(i) $Mag(A_\mu) > Mag(B_\mu)$ if and only if $A \succ B$,

(ii) $Mag(A_\mu) < Mag(B_\mu)$ if and only if , $A \prec B$,

(iii) $Mag(A_\mu) = Mag(B_\mu)$ and $Mag(A_\gamma) = Mag(B_\gamma)$ if and only if $A \sim B$,

(iv) $Mag(A_\mu) = Mag(B_\mu)$ and $-Mag(A_\gamma) > -Mag(B_\gamma)$ if and only if $A \succ B$.

(v) $Mag(A_\mu) = Mag(B_\mu)$ and $-Mag(A_\gamma) < -Mag(B_\gamma)$ if and only if $A \prec B$.

Then the order $\succeq$ and $\preceq$ is formulated as A $\succeq$ B if and only if A $\succ$ B or , A $\preceq$ B if and only if A $\prec$ B or In the otherwords this method is placed in the first class of Kerre's categories[5]

4.Numerical examples

Example 1

Consider the two TIFNs A = (3,5,7,9,12,14,16,18) and B =(4,6,7,11,13,15,18,20) .

Step 1 : $Mag(A_\mu)$ = 10.5 and $Mag(B_\mu)$ = 12 using(6).

Since $Mag(A_\mu) < Mag(B_\mu)$, the ranking order is A $\prec$ B.

Example 2

Consider the two TIFNs A = (3,5,7,9,12,14,16,18) and B = (4,5,7,9,12,15,16,20)

Step 1 : $Mag(A_\mu)$ = 10.5 and $Mag(B_\mu)$ = 10.5 using(6).

Since $Mag(A_\mu) = Mag(B_\mu)$, so go to step 2.

Step 2 : $Mag(A_\gamma)$ = 10.5 and $Mag(B_\gamma)$ = 11.17. Since $Mag(B_\gamma) \neq Mag(A_\gamma)$, -Mag() = -10.5

and $-Mag(B_\gamma)$ = -11.17. Hence the ranking order is A $\succ$ B

Example 3

Consider the two TIFNs A = (3,5,7,9,12,14,16,18) and B = (1,5,7,9,12,14,16,20).

Step 1: $Mag(A_\mu)$ = 10.5 and $Mag(B_\mu)$ = 10.5. Since $Mag(A_\mu) = Mag(B_\mu)$, so go to Step 2.

Step 2: $Mag(A_\gamma)$ = 10.5 and $Mag(B_\gamma)$ = 10.5. Since $Mag(A_\gamma) = Mag(B_\gamma)$,

the ranking order is A ~ B

These sets of TIFNs are compared with the existing approaches in Table 1.

Table 1. A Comparison of the ranking results for different approaches.

Example	Hassan Mismmast Nehi Method [27]	Grzegorzewski Method[19]	Proposed Method
1	A $\prec$ B	A $\prec$ B	A $\prec$ B
2	A $\succ$ B	A $\prec$ B	A $\succ$ B
3	~	~	~

5.Conclusion

Here, magnitude of membership function and non-membership function of an IFN are introduced. Based on this, a new method for ranking of TIFN is proposed. To illustrate the new ranking method, numerical examples are provided and compared with existing approaches proposed by Hassan [20] and Grzegorzewski [12].This approach can be applied to order the TIFN in solving different fuzzy optimization problems.

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